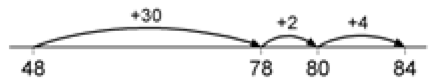
St John's Calculation Policy

Parent's Guide

If what you remember as maths is pages of sums you may sometimes feel confused when your child's maths book contains writing, pictures, diagrams, jottings or blank number lines and not many 'formal calculations'.

We know that school wide guidance, such as this, can ensure consistency of approach, enabling children to progress stage by stage through models and representations they recognise from previous teaching, allowing for deeper conceptual understanding and fluency. As children move between concrete and pictorial and abstract, teachers will be presenting strategies and equipment appropriate to children's level of understanding. the aim is that by the end of Key Stage 2, the great majority of children should be able to use an efficient written method for each operation with confidence and understanding. This guidance promotes the use of what are commonly known as 'standard' written methods – methods that are efficient and work for any calculations, including those which involve whole numbers or decimals.

Year Six

Stage 1 – The empty number line	
• The mental methods that lead to column addition generally involve partitioning e.g. adding the tens and units separately. Children also need to be able to partition numbers in ways other than into tens and units to help them make multiples of ten by adding in steps. • The empty number line helps to record the steps on the way to calculating the total.	Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10. 8 + 7 = 15  48 + 36 = 84  or: 
Stage 2 – Partitioning	
• The next stage is to record mental methods using partitioning. Add the tens and then the units to form partial sums and then add these partial sums.	47 + 76 = 40 + 70 = 110 7 + 6 = 13 110 + 13 = 123
Stage 3 – Expanded method in columns	
• Move on to a layout showing the addition of the tens to the tens and the units to the units separately. To find the partial sums either the tens or the units can be added first, and the total of the partial sums can be found by adding them in any order. **As children gain confidence, ask**	Adding the tens first: 47 +76 110 13 123 **Discuss how adding the units first gives the same answer as adding the tens first. Refine over time to adding the units digits first consistently.**

- **them to start by adding the units digits first always.**
- The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Adding the units first:

```
  47
 +76
 ---
  13
 110
 ---
 123
```

Stage 4 – Column method

- In this method, recording is reduced further. Exchange digits are recorded **below** the line, using the words 'exchange ten' or 'exchange one hundred', **not** 'exchange one'.
- Later, extend to adding three two-digit numbers, two three-digit numbers and numbers with different amounts of digits.

```
   47     258    366
 + 76   +  87   +458
  123     345    824
  11      11     11
```

Subtraction

Stage 1 – The empty number line

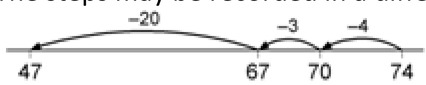
- The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74-27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten.
- The steps can also be recorded by counting up from the smaller to the larger number to find the difference.
- With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57-12, 86-77 or 43-28.

Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10.

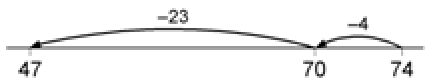
15 - 7 = 8

74 - 27 = 47 worked by counting back:

The steps may be recorded in a different order:



or combined:

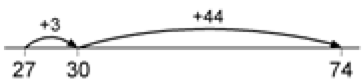


Stage 2 The counting – up method

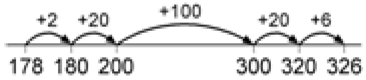
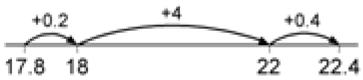
- The mental method of counting up from the smaller to the larger number can be recorded using either number lines or vertically in columns. The number of rows (or steps) can be reduced by combining steps. With two-digit numbers, this requires children to be able to work out the answer to a calculation such as 30 + ? = 74 mentally.


```
  74
- 27
   3   → 30
  40   → 70
   4   → 74
  47
```

Or:




```
  74
- 27
   3   → 30
  44   → 74
  47
```

• With three-digit numbers the number of steps can again be reduced, provided that children are able to work out answers to calculations such as 178 + ? = 200 and 200 + ? = 326 mentally. • The most compact form of recording remains reasonably efficient.	 326 −178 2 → 180 20 → 200 100 → 300 26 → 326 148 Or:  326 −178 22 → 200 126 → 326 148
• The method can be used with decimals where no more than three columns are required. However, it becomes less efficient when more than three columns are needed. • This counting-up method can be a useful alternative for children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.	 22.4 −17.8 0.2 → 18 4.0 → 22 0.4 → 22.4 4.6 Or:  22.4 −17.8 0.2 → 18 4.4 → 22.4 4.6
Stage 3 – Partitioning	
• Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For 74 - 27 this involves partitioning the 27 into 20 and 7, and then subtracting from 74 the 20 and the 4 in turn. Some children may need to partition the 74 into 70 + 4 or 60 + 14 to help them carry out the subtraction.	74 – 27 = 74 – (20 + 7) 74 – 20 = 54 54 – 7 = 47
Stage 4 – Expanded method in columns	
• Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where ones are placed under ones and tens under tens. • This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills. • The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.	Partitioned numbers are then written under one another: Example: 74 - 27 $\begin{array}{r} 70+4 \\ -\ 20+7 \\ \hline \end{array}$ $\begin{array}{r} \overset{60}{\cancel{70}}+\overset{14}{\cancel{4}} \\ -\ 20+7 \\ \hline 40+7 \end{array}$ $\begin{array}{r} \overset{6}{\cancel{7}}\ \overset{14}{\cancel{4}} \\ -\ 2\ 7 \\ \hline 4\ 7 \end{array}$ Example: 741 - 367 $\begin{array}{r} 700+40+1 \\ -\ 300+60+7 \\ \hline \end{array}$ $\begin{array}{r} \overset{600}{\cancel{700}}+\overset{130}{\cancel{40}}+\overset{11}{\cancel{1}} \\ -\ 300+60+7 \\ \hline 300+70+4 \end{array}$ $\begin{array}{r} \overset{6}{\cancel{7}}\ \overset{13}{\cancel{4}}\ \overset{11}{\cancel{1}} \\ -\ 3\ 6\ 7 \\ \hline 3\ 7\ 4 \end{array}$
The expanded method for three – digit numbers	

Example: 563 − 241, no adjustment or decomposition needed

Expanded method

$$\begin{array}{r} 500+60+3 \\ -\ 200+40+1 \\ \hline 300+20+2 \end{array}$$

leading to

$$\begin{array}{r} 563 \\ -\ 241 \\ \hline 322 \end{array}$$

Start by subtracting the ones, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying 'sixty take away forty', not 'six take away four'.

Example: 563 − 271, adjustment from the hundreds to the tens, or partitioning the hundreds

$$\begin{array}{r} 500+60+3 \\ -\ 200+70+1 \\ \hline \end{array} \quad \begin{array}{r} 400+160+3 \\ -\ 200+70+1 \\ \hline 200+90+2 \end{array} \quad \begin{array}{r} \overset{400}{\cancel{500}}+\overset{160}{\cancel{60}}+3 \\ -\ 200+70+1 \\ \hline 200+90+2 \end{array} \quad \begin{array}{r} \overset{4}{\cancel{5}}\,\overset{16}{\cancel{6}}\,3 \\ -\ 2\,7\,1 \\ \hline 2\,9\,2 \end{array}$$

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixty-three'. Then discuss the hundreds, tens and ones components of the number, and how 500 + 60 can be partitioned into 400 + 160. The subtraction of the tens becomes '160 minus 70', an application of subtraction of multiples of ten.

Example: 563 − 278, adjustment from the hundreds to the tens and the tens to the ones

$$\begin{array}{r} 500+60+3 \\ -\ 200+70+8 \\ \hline \end{array} \quad \begin{array}{r} 400+150+13 \\ -\ 200+70+8 \\ \hline 200+80+5 \end{array} \quad \begin{array}{r} \overset{400}{\cancel{500}}+\overset{150}{\cancel{60}}+\overset{13}{\cancel{3}} \\ -\ 200+70+8 \\ \hline 200+80+5 \end{array} \quad \begin{array}{r} \overset{4}{\cancel{5}}\,\overset{15}{\cancel{6}}\,\overset{13}{\cancel{3}} \\ -\ 2\,7\,8 \\ \hline 2\,8\,5 \end{array}$$

Here both the tens and the ones digits to be subtracted are bigger than both the tens and the ones digits you are subtracting from. Discuss how 60 + 3 is partitioned into 50 + 13, and then how 500 + 50 can be partitioned into 400 + 150, and how this helps when subtracting.

Example: 503 − 278, dealing with zeros when adjusting

$$\begin{array}{r} 500+0+3 \\ -\ 200+70+8 \\ \hline \end{array} \quad \begin{array}{r} 400+90+13 \\ -\ 200+70+8 \\ \hline 200+20+5 \end{array} \quad \begin{array}{r} \overset{400}{\cancel{500}}+\overset{90}{\cancel{0}}+\overset{13}{\cancel{3}} \\ -\ 200+70+8 \\ \hline 200+20+5 \end{array} \quad \begin{array}{r} \overset{4}{\cancel{5}}\,\overset{9}{\cancel{0}}\,\overset{13}{\cancel{3}} \\ -\ 2\,7\,8 \\ \hline 2\,2\,5 \end{array}$$

Here 0 acts as a place holder for the tens. The adjustment has to be done in two stages. First the 500 + 0 is partitioned into 400 + 100 and then the 100 + 3 is partitioned into 90 + 13.

Stage 5 – Column method

- This compact method follows from the expanded column method. When exchanging over make sure you refer to 'exchanging ten' or 'exchanging hundred' not exchanging one. The exchanged digit should be written above the original digit.

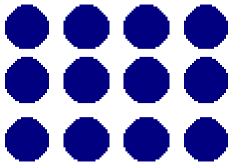
$$\begin{array}{r} \overset{6}{\cancel{7}}\,\overset{14}{\cancel{4}} \\ -\ 2\,7 \\ \hline 4\,7 \end{array}$$ TU - TU

$$\begin{array}{r} \overset{4}{\cancel{5}}\,\overset{15}{\cancel{6}}\,\overset{13}{\cancel{3}} \\ -\ 2\,7\,8 \\ \hline 2\,8\,5 \end{array}$$ HTU - HTU

Progression in Multiplication and Division

Multiplication and division are connected.
Both express the relationship between a number of equal parts and the whole.

Part	Part	Part	Part
Whole			

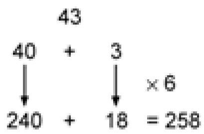


Multiplication

Stage 1 – Mental multiplication using partitioning

- Mental methods for multiplying TU × U can be based on the distributive law of multiplication over addition. This allows the tens and ones to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the ones can be multiplied first but it is more common to start with the tens.

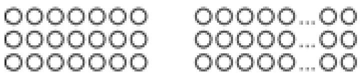
Informal recording in Year 4 might be:



Also record mental multiplication using partitioning:

$$14 \times 3 = (10 + 4) \times 3$$
$$= (10 \times 3) + (4 \times 3) = 30 + 12 = 42$$
$$43 \times 6 = (40 + 3) \times 6$$
$$= (40 \times 6) + (3 \times 6) = 240 + 18 = 258$$

Note: These methods are based on the distributive law. Children should be introduced to the principle of this law (not its name) in Years 2 and 3, for example when they use their knowledge of the 2, 5 and 10 times-tables to work out multiples of 7:



$$7 \times 3 = (5 + 2) \times 3 = (5 \times 3) + (2 \times 3) = 15 + 6 = 21$$

Stage 2 – The grid method

- As a staging post, an expanded method which uses a grid can be used. This is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps.
- It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.

38 × 7 = (30 × 7) + (8 × 7) = 210 + 56 = 266

×	7
30	210
8	56
	266

- The next step is to move the number being multiplied (38 in the example shown) to an extra row at the top. Presenting the grid this way helps children to set out the addition of the partial products 210 and 56.
- The grid method may be the main method used by children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.

	30 + 8
×	7
	210
	56
	266

Stage 3 – Expanded short multiplication

- The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above.
- Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is 'thirty multiplied by seven', not 'three times seven', although the relationship 3 × 7 should be stressed.
- Most children should be able to use this expanded method for TU × U by the end of Year 4.

```
30 + 8                         38
×    7                        ×  7
   210    30 × 7 = 210         210
    56     8 × 7 =  56          56
   266                         266
```

Stage 4 – Short multiplication

- The recording is reduced further, with carry digits recorded below the line.
- If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 3.

```
 38
× 7
266
 5
```

The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage.

Stage 6 – Two-digit by two-digit products

- Extend to TU × TU, asking children to estimate first.
- Start with the grid method. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product.
- As in the grid method for TU × U in stage 4, the first column can become an extra top row as a stepping stone to the method below.

56 × 27 is approximately 60 × 30 = 1800.

×	20	7	
50	1000	350	1350
6	120	42	162
			1512
			1

	50	6	
×	20	7	
	1000	350	1350
	120	42	162
			1512
			1

- Reduce the recording, showing the links to the grid method above.

56 × 27 is approximately 60 × 30 = 1800.

```
  56
x 27
  42    6 x 7 = 42
 350    50 x 7 = 350
 120    6 x 20 = 120
```

• Reduce the recording further. • The carry digits in the partial products of 56 × 20 = 120 and 56 × 7 = 392 are usually carried mentally. • The aim is for most children to use this long multiplication method for TU × TU by the end of Year 5.	56 × 27 is approximately 60 × 30 = 1800. 56 x 27 392 56 x 7 1120 56 x 20
Stage 7 – Three-digit by two-digit products	
• Extend to HTU × TU asking children to estimate first. Start with the grid method. • It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.	286 × 29 is approximately 300 × 30 = 9000. <table><tr><td>×</td><td>20</td><td>9</td><td></td></tr><tr><td>200</td><td>4000</td><td>1800</td><td>5800</td></tr><tr><td>80</td><td>1600</td><td>720</td><td>2320</td></tr><tr><td>6</td><td>120</td><td>54</td><td>174</td></tr><tr><td></td><td></td><td></td><td>8294</td></tr><tr><td></td><td></td><td></td><td>1</td></tr></table>
Reduce the recording, showing the links to the grid method above. • This expanded method is cumbersome, with six multiplications and a lengthy addition of numbers with different numbers of digits to be carried out. There is plenty of incentive to move on to a more efficient method.	286 x 29 54 6 x 9 = 54 720 80 x 9 = 720 1800 200 x 9 = 1800 120 6 x 20 = 120
• Children who are already secure with multiplication for TU × U and TU × TU should have little difficulty in using the same method for HTU × TU. • Again, the carry digits in the partial products are usually carried mentally.	286 × 29 is approximately 300 × 30 = 9000. 286 x 29 2574 286 x 9 5720 286 x 20

Division

Stage 2 – Mental division using partitioning	
• Mental methods for dividing TU ÷ U can be based on partitioning and on the distributive law of division over addition. This allows a multiple of the divisor and the remaining number to be divided separately. The results are then added to find the total quotient.	One way to work out TU ÷ U mentally is to partition TU into a multiple of the divisor plus the remaining ones, then divide each part separately.

- Many children can partition and multiply with confidence. But this is not the case for division. One reason for this may be that mental methods of division, stressing the correspondence to mental methods of multiplication, have not in the past been given enough attention.
- Children should also be able to find a remainder mentally, for example the remainder when 34 is divided by6.

Informal recording in Year 4 for 84 ÷ 7 might be:

$$\begin{array}{ccccl} & 84 & & & \\ 70 & + & 14 & & \\ \downarrow & & \downarrow & \div 7 & \\ 10 & + & 2 & = 12 & \end{array}$$

In this example, using knowledge of multiples, the 84 is partitioned into 70 (the highest multiple of 7 that is also a multiple of 10 and less than 84) plus 14 and then each part is divided separately using the distributive law.

Another way to record is in a grid, with links to the grid method of multiplication.

×		
7	70	14

→

×	10	2
7	70	14

10 + 2 = 12

As the mental method is recorded, ask: 'How many sevens in seventy?' and: 'How many sevens in fourteen?'

Also record mental division using partitioning:

$$\begin{aligned} 64 \div 4 &= (40 + 24) \div 4 \\ &= (40 \div 4) + (24 \div 4) \\ &= 10 + 6 = 16 \end{aligned}$$

$$\begin{aligned} 87 \div 3 &= (60 + 27) \div 3 \\ &= (60 \div 3) + (27 \div 3) \\ &= 20 + 9 = 29 \end{aligned}$$

Remainders after division can be recorded similarly.

$$\begin{aligned} 96 \div 7 &= (70 + 26) \div 7 \\ &= (70 \div 7) + (26 \div 7) \\ &= 10 + 3 \text{ R } 5 = 13 \text{ R } 5 \end{aligned}$$

Stage 3 – Short division of TU by U

- 'Short' division of TU ÷ U can be introduced as a more compact recording of the mental method of partitioning.
- Short division of two-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound.
- For most children this will be at the end of Year 4 or the beginning of Year 5.
- The accompanying patter is 'How many threes divide into 80 so that the answer is a multiple of 10?' This gives 20 threes or 60, with 20 remaining. We now ask: 'What is 21divided by three?' which gives the answer 7.

For 81 ÷ 3, the dividend of 81 is split into 60, the highest multiple of 3 that is also a multiple 10 and less than 81, to give 60 + 21.Each number is then divided by 3.

$$\begin{aligned} 81 \div 3 &= (60 + 21) \div 3 \\ &= (60 \div 3) + (21 \div 3) \\ &= 20 + 7 \\ &= 27 \end{aligned}$$

> This stage need only be discussed and modelled as a whole class input. There is no need for the children to record like this unless you find it beneficial for your group.

The short division method is recorded like this:

$$\begin{array}{r} 2\ 7 \\ 3\overline{)8\ {}^{2}1} \end{array}$$

	The carry digit '2' represents the 2 tens that have been exchanged for 20 ones. It is written as a superscript in front of the 1 to show that 21 is to be divided by 3. The 27 written above the line represents the answer:
Stage 4 – Short division of HTU by U	
• 'Short' division of HTU ÷ U can be introduced as a more compact recording. No chunking is involved since the links are to partitioning, not repeated subtraction. • The accompanying pattern is 'How many threes in 290?' (the answer must be a multiple of 10). This gives 90 threes or 270, with 20 remaining. We now ask: 'How many threes in 21?' which has the answer 7. • Short division of a three-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound. • For most children this will be at the end of Year 5 or the beginning of Year 6.	For 291 ÷ 3, because 3 × 90 = 270 and 3 × 100 = 300, we use 270 and split the dividend of 291 into 270 + 21. Each part is then divided by 3. $\begin{aligned} 291 \div 3 &= (270 + 21) \div 3 \\ &= (270 \div 3) + (21 \div 3) \\ &= 90 + 7 \\ &= 97 \end{aligned}$ *This stage need only be discussed and modelled as a whole class input. There is no need for the children to record like this unless you find it beneficial for your group.* The short division method is recorded like this: $3\overline{)290+1} = \begin{array}{r} 90+7 \\ 3\overline{)270+21} \end{array}$ This is then shortened to: $\begin{array}{r} 9\;7 \\ 3\overline{)2\;9\;{}^{2}1} \end{array}$ The carry digit '2' represents the 2 tens that have been exchanged for 20 ones. In the first recording above it is written in front of the 1 to show that a total of 21 ones are to be divided by 3. The 97 written above the line represents the answer: 90 + 7, or 9 tens and 7 ones.
Stage 5 – Long division of HTU by TU	
The next step is to tackle HTU ÷ TU, which for most children will be in Year 6. The layout on the right, which links to chunking, is in essence the 'long division' method. Recording the build-up to the quotient on the left of the calculation keeps the links with 'chunking' and reduces the errors that tend to occur with the positioning of the first digit of the quotient. Conventionally the 20, or 2 tens, and the 3 ones forming the answer are recorded above the line, as in the second recording.	How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560. $\begin{array}{rrl} & 24\overline{)\,560} & \\ 20 & -\underline{480} & 24 \times 20 \\ & 80 & \\ 3 & \underline{72} & 24 \times 3 \\ & 8 & \end{array}$ Answer: 23 R 8 In effect, the recording above is the long division method, though conventionally the digits of the answer

	are recorded above the line as shown below. 23 24) 560 −480 80 −72 8 Answer: 23 R 8

What you can do as parents:

- Have a look at the strategies we use in order to help with homework ina way the children are familiar with;
- Rehearse number facts often and thoroughly. These are the basis of most calculations and need to be learnt in order to be built on. It does take time, patience and practice at home but it leads to quicker calculating and the confidence of recognising something familiar;
- Practise couting forwards and backwards to 100 from any numberin steps of 2,3,4,5,8,10,50 and 100
- Learn number bonds to one hundred and one thousand (multiples of 10);
- Rehearse multiplication facts - by the end of Year 4 children are expected to know all multiplication facts up to 12 x 12.
- Play number based games like cards or board games. Access websites to encourage reasoning skills like http://nrich.maths.org/students or http://resources.woodlands.kent.sch.uk/maths/index.html